Type Theories as Foundational Languages*

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In this talk, I'll introduce modern type theories, as studied by Martin-Löf and others [9, 2, 5, 7], and discuss their employment as foundational languages. Usually, type theories would need be extended to be suitable foundational languages in various applications and, in particular, the notion of equality need be carefully studied. For example, the univalence axiom and higher inductive types [3, 4] are studied in univalent foundations of mathematics [10, 3], where type isomorphism is regarded as the intended equality. In type-theoretical semantics [1, 7], where the traditional notion of equality can adequately be employed, a suitable subtyping mechanism [6, 8] is needed, among other things. My talk will be informal and, if time permits, I'll also discuss some ongoing research topics, albeit only briefly.

References

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